Iterative Integration of Dynamic Data in Reservoir Models
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Abstract
Conditioning reservoir models to dynamic data such as historical production response is challenging because of the complexity of the relationship between the model parameters and the response variable. The focus of this paper is to present a methodology for efficiently integrating dynamic production data into reservoir models. In contrast to some of the other methods, the proposed methodology attempts to quantify the information in production data pertaining to reservoir heterogeneity in a probabilistic manner. The conditional probability representing the uncertainty in permeability at a location is iteratively updated to account for the additional information contained in the dynamic response data. A localized perturbation procedure is also presented to account for multiple flow regions within the reservoir. Such an improved scheme utilizes a set of locally varying deformation parameters to guide the iterative updating process in order to obtain a global history match.

Introduction
Geostatistics provides a framework for integrating diverse types of reservoir specific data in order to develop multiple realizations of the reservoir. The acquired data may display spatial dependency only (static data) such as well logs, core measurements, etc. or may display joint space-time dependency (dynamic data) such as production response, time-lapse seismic, etc. Several algorithms are available to condition reservoir models to static data, however conditioning to dynamic data is complex because of the non-linear relationship between the input model parameters (spatially varying petrophysical properties) and the output response (e.g. well pressure as a function of time). Manually adjusting reservoir models so that they reflect the dynamic response information accurately is very time consuming and tedious. In addition, such manual adjustments might lead to reservoir models that do not exhibit the correct spatial covariance structure. Consequently, though the adjusted reservoir models may display an excellent match to the historical production records, they may yield totally erroneous future predictions of reservoir performance. The ability to forecast future production scenarios accurately is the ultimate objective of any reservoir modeling exercise.

This problem could be alleviated if the historical dynamic data are integrated into the reservoir model construction step such that the final model is conditioned to all the available static data as well as the dynamic data. Provided the rules for integrating production information into geologic models can be clearly established through calibration, incremental information derived from production data collected during the productive life of the field can be used to continuously update reservoir models.

The construction of permeability fields constrained to dynamic well data can be treated as a classical, ill-posed inverse problem. Stochastic optimization techniques1,II have been proposed that make use of sensitivity coefficientsIII to perturb the model parameters repeatedly. The calculation of these sensitivity coefficients requires frequent runs of a flow simulator which is computationally expensive. The pilot point methodIV, V seeks to reduce the dimensionality of the stochastic optimization method by performing perturbations at only certain control points and then spreading these perturbations throughout the reservoir domain using some spatial interpolation scheme such as kriging. However the selection of control points remains arbitrary and the resultant reservoir models are influenced strongly by the choice of pilot points. In addition the inversion process remains computationally expensive for field scale history matching. Markov chain Monte Carlo (MCMC) methods have been applied in earth sciences for modeling reservoirs VI, VII. MCMC is an iterative method where one generates a Markov chain forward in time, and this chain eventually converges to the desired stationary probability distribution. Algorithmically, each iterative step consists of a proposal and an acceptance step. The acceptance probability can be related to the ratio of likelihood of observing the dynamic data given the new updated value of permeability to the likelihood with the old value VIII. However the calculation of these likelihood probabilities again require repeated execution of a flow simulator rendering the procedure cpu expensive. Srinivasan, S. VIII, showed that it is possible to reduce the computation time by utilizing multi-

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point proxy functions as a substitute for the full flow simulator in order to calibrate the required likelihood functions. The multi-point proxy captures the underlying non-linear relationship between the input permeability field and the output response variables. Gradual deformation methods have recently become popular for constraining multi-Gaussian random fields to dynamic data. The basic principle of the gradual deformation method is to generate realizations that evolve smoothly at each step so as to honor specific constraints posed by the prior structural model, the seismic data and/or the production data. The major disadvantage is that the procedure can be viably applied only for multi-Gaussian fields.

The intent of this paper is to introduce a dynamic data integration algorithm that is computationally fast, does not suffer from any multi-Gaussian restrictions and hence can be even applied to reservoirs exhibiting severe discontinuities such as facies/indicator type distributions.

Proposed Iterative methodology

The objective of developing a reservoir model that is conditioned to the “hard-data” and the dynamic production data could be achieved by generating the joint conditional probability $P(A|B,C)$ where $A$ is the simulation event ($k(u)$, $u \in$ Reservoir), $B$ is the information from the “hard-data” and $C$ is the information from the production data. If the conditional distribution is modeled accurately, any realization sampled from that probability distribution would display the correct static and dynamic characteristics. Employing a permanence of ratio hypothesis, the conditional probability $P(A|B,C)$ can be expressed in terms of $P(A)$, $P(A|B)$ and $P(A|C)$. The prior probability $P(A)$ is known given the hard data and assuming stationarity, the probability $P(A|B)$ can be derived by kriging utilizing the model for spatial covariance. The probability $P(A|C)$ relates the permeability field to the dynamic data and is difficult to compute directly and consequently an iterative calibration procedure was implemented. Consider a Markov chain comprising of iterative steps $\ell = 1, ..., L$ such that the outcome of the indicator RV $I(u)$ at step $\ell + 1$ is dependent only on the outcome at the previous step $\ell$. The chain is parameterized by a dynamic factor $r_D \in [0,1]$ that quantifies the probability of transitioning from indicator category $k$ at step $\ell$ to the category $k'$ at step $\ell + 1$ given the historic production data.

The parameterization is written as:

$$P \left[ I^{\ell+1}(u) = k' \mid I^{\ell}(u) = k, C \right] = r_D \cdot P[I(u) = k'] \quad \forall k' \neq k$$

$$P \left[ I^{\ell+1}(u) = k \mid I^{\ell}(u) = k, C \right] = 1 - \sum_{k' \neq k} r_D \cdot P[I(u) = k']$$

The probability $P[I(u) = k']$ is the same as the prior probability $P(A)$ and represents the marginal probability corresponding to class $k'$ while the left hand side of the above formulation is the conditional probability $P(A|C)$ that is being sought. A higher value of $r_D$ implies a higher probability for the current category $k$ to change to a new category $k'$ from step $\ell$ to step $\ell + 1$. As a result the realization sampled at the step $\ell + 1$ may be significantly different from that at step $\ell$. Conversely, a low value of $r_D$ signifies a higher probability to remain in the same category $k$ over the step. This gives rise to a realization that is quite similar to that at the previous step.

Starting with an initial guess for $r_D$, the probability $P(A|C)$ corresponding to different indicator categories $k = 1, ..., K + 1$ is calculated. This probability is combined with $P(A|B)$ obtained from indicator kriging using the permanence of ratio hypothesis in order to derive the joint conditional probability $P(A|B,C)$. The indicator category at the node $u$ is obtained by sampling from $P(A|B,C)$. The reservoir model obtained at this iteration step is then flow simulated. The deviation of the simulated production response from the target response is expressed in the form of an objective function:

$$\Delta O(r_D) = \left\| f_{\text{target}}(t) - f^*(r_D, t) \right\|^2$$

The objective function is minimized by iteratively adjusting the deformation parameter $r_D$. A 1-D, non-gradient based optimization procedure such as the Dekker-Brent method is utilized to obtain the optimal value of the parameter $r_D$. The updated realization $I^{\ell+1}(u)$ is obtained corresponding to the optimal $r_D$ value. The iterations $\ell = 1, ..., L$ are performed until a global match to the production history data is obtained. The algorithm described above has been applied successfully to history-match several 2-D and 3-D reservoir examples.

The effect of various variables such as type of history-match variables and length of production records was studied in that reference.

Multiple Reservoir Domains

The spatial connectivity patterns displayed by the reservoir could be dissimilar in different regions within the reservoir. In such a case, it may be required that the permeability distributions in different regions of the reservoir would have to be adjusted differently in order to accommodate the constraints posed by the dynamic data. If a specific well pair shows a response very different from the field data, the reservoir volume in the vicinity of that well pair should be changed more drastically than well-pairs for which the match is already good. The dynamic parameter $r_D$ controls the transitions in the permeability values under the influence of the dynamic constraints. Hence the region of the reservoir showing a severe departure from the historical production records might require a locally higher value of $r_D$. This creates a need for a localized perturbation scheme. Another incentive for a localized scheme would be when dealing with reservoir scenarios involving multiple wells that are brought on line in a phased manner. It may be computationally inefficient in such cases to adjust the entire reservoir model to obtain a history-match and it may be more appropriate to perform perturbations to the probability models locally in regions where the additional production information becomes available. Since the geological constraints are always honored via the conditional probability $P(A|B)$, the reservoir models constructed using such local perturbations would not exhibit any artifact discontinuities.
a) Definition of Domains

A localized perturbation scheme would require the delineation of multiple reservoir domains. These zones could be defined on the basis of various parameters such as sensitivity coefficients, pressure gradients, streamline density, etc. In this work, streamline density was used to define the zones.

In streamline simulation the solution to the governing flow equations is obtained along 1-D streamlines which are orthogonal to the isopotential lines. The major assumption made is that flow is primarily convective whereby capillary and dispersive flow effects are ignored. The saturation profile along each streamline is obtained and these are then spatially interpolated to obtain saturations at all locations within the reservoir. The number of streamlines passing through a particular grid block i.e. the streamline density is proportional to the volume flux of fluids through that block. Regions that have a high density of streamlines indicate permeability zones that have a crucial impact on the flow response corresponding to a well pair and should be adjusted to get a history match. The streamlines may be sorted according to the origin and destination cells thereby making it possible to assign well pairs to the streamlines. Thus regions contributing to flow for that particular well pair can be identified. It may be desirable to only select those cells that are relatively more important to flow as compared to the others. In such a case a minimum streamline density could be imposed on all the cells and an aggregate of all cells whose streamline density is above the threshold can be assumed to define a reservoir region that influences the response corresponding to the particular well pair. This threshold density must be considered on a case-by-case basis by performing some sensitivity runs. The streamline simulator used for this research is 3DSL© (Streamsim Technologies Inc.)XVIII. The software offers a feature for writing out the coordinates of the cells through which the streamline passes and the coordinates of the well pair to which each streamline belongs. A post-processing program was written in order to read the output file from 3DSL and then assign an indicator code to the cells through which a particular streamline passes.

b) Iterative Updating of Multiple Regions

In the global perturbation scheme, the deformation of the reservoir model due to the dynamic data is controlled by a single \( r_D \) parameter. However in the localized case, the modification of the probability distributions would be controlled by multiple \( r_D \) parameters, say 1 for each region. In such a case, optimization would have to be performed with respect to each of the dynamic parameters. This could be achieved with a multi-dimensional optimization scheme to jointly optimize the set \( r_D \) comprising of \( n \) deformation parameters. However, such multiparameter optimization may add significantly to the computational burden and consequently a sequential 1-dimensional optimization procedure was implemented as outlined below.

The optimization procedure was performed such that at any iteration step the value of the dynamic parameter \( r_D \) is only updated in a single region. The value of \( r_D \) for the other regions is fixed at the last best value obtained for them. The inner loop of the algorithm is thus split into different sub-loops, one for each region. Within each sub-loop, the optimization is carried out for only one of the regions with the Dekker-Brent method. Values of \( r_D \) required to generate the complete reservoir model are maintained fixed at the previous optimal values. After, the first optimal \( r_D \) value is established the optimization of the value for the second region is carried out using the updated \( r_D \) value for the first region and the initial \( r_D \) values for the remaining regions. This procedure is explained in the flow chart shown in Figure 1. In all regions where either no streamline passes through or the streamline density is less than the minimum density specified, the value of \( r_D = 0 \) and consequently no perturbation to the conditional probability distribution is performed in those regions. The reservoir property in these regions is obtained by sampling from the conditional distributions \( P(A|B) \) obtained by indicator kriging.

Convergence Issues

The convergence properties of the algorithm were studied on a 3-D (50x50x3) synthetic reservoir model. A line drive pattern consisting of one injector well and two producer wells was flow simulated. The investigations were performed by applying the global perturbation scheme wherein the entire reservoir model is updated using a single \( r_D \) parameter. The histogram of the target reservoir model shown in Figure 2 is representative of a non-gaussian, indicator type field. The change in objective function with iterations is shown in Figure 3. As may be seen even though there is a general decreasing trend exhibited by the objective function there are persistent fluctuations. The saw-tooth behavior of the objective function can add to the computational cost. The observed saw tooth profile can be ascribed to the abrupt permeability transitions in a indicator random field as opposed to the smoother transitions in a Gaussian permeability field. In order to validate this point, a Gaussian permeability field was developed in order to make a comparison with a indicator field. The histogram of the field shown in Figure 4 has a mean of about 500md and std. dev. of about 250md. The same flow simulation parameters were used as in the indicator case and the algorithm was applied. The objective function behavior is shown in Figure 5. The objective function in this case is devoid of the oscillations seen in the earlier case and decreases smoothly.

The uncontrolled fluctuations of the objective function can thus be explained in terms of the irregular, non-Gaussian characteristics of the underlying conditional and marginal probability distributions. In the case of Gaussian distributions, permeability quantiles sampled from the distributions vary smoothly implying that variations in the permeability model corresponding to variations in the value of \( r_D \) are smooth and the resulting objective function value also varies smoothly. However in the indicator field case, even for low values of the parameter \( r_D \) the joint conditional distributions can be altered significantly (because the probabilities are computed only on a limited set of thresholds) thereby resulting in significantly different realizations. Flow simulation on these vastly varying realizations can lead to very dissimilar production responses that are manifested as an oscillation of the objective function. Several attempts were made to develop schemes that dampen the oscillations of the objective function XIX.
Rejection-Sampling Scheme
The oscillating nature of the objective function suggests that the algorithm might give rise to a realization \( I^{\ell+1}(u) \) that is considerably different from the realization \( I^{\ell}(u) \) at the end of the previous optimization loop. This realization \( I^{\ell+1}(u) \) could be an adverse realization (from a flow perspective), that could eventually cause the optimization process to drift away from the global optimum. Consequently, we propose a rejection scheme in which the objective function at the start of each Dekker-Brent loop is checked to verify that it is lower than the lowest objective function in the preceding outer iteration. In the event, the objective function value is higher the iterative procedure is repeated starting from a different initialization. Hence the procedure screens random seeds continuously until a suitable random seed that ultimately leads to convergence of the procedure is selected. Such a rejection scheme does result in an increase in cpu time. However, the gain in convergence speed and robustness gained by adopting such an approach are likely to far outweigh the additional computational expense.

The modified rejection-sampling scheme was implemented for the test case and the results are shown in Figure 6. The objective function shows a well-behaved monotonic decrease.

Realistic Reservoir example
A 3-dimensional (50 x 50 x 5) synthetic synclinal reservoir model was carefully generated for the purpose of demonstrating the localized perturbation scheme. The reservoir was chosen to have a synclinal structure with unequal dips on both the flanks. The structure of the reservoir is shown in Figure 7. The location of the wells in an inverse spot pattern is also shown in the figure. The individual layers of the target reservoir model are shown in Figure 8. The producers each operate at a rate of 1800bbls/d and the water injector I-1 is assumed to operate at 7200bbls/d. The remaining model parameters specified are shown in Table 1. The structural configuration of the reservoir and the spatial variability of the permeability field render this case example amenable to the process of segregating the reservoir into multiple flow regions on the basis of streamline density.

A first-step streamline simulation was performed and the traced streamlines were sorted according to the injector-producer pair to which they connect. The grid blocks were indicator coded using the following scheme,

\[
I'(u) = \begin{cases} 
1 & \text{if (streamline } \in \text{ I-1 and } P-1) \\
2 & \text{if (streamline } \in \text{ I-1 and } P-2) \\
3 & \text{if (streamline } \in \text{ I-1 and } P-3) \\
4 & \text{if (streamline } \in \text{ I-1 and } P-4) \\
0 & \text{otherwise} 
\end{cases}
\]

The flow regions corresponding to the reference reservoir are shown in Figure 9. The figure only shows the topmost layer of the reservoir. Similar regions were traced for all other layers of the 5-layered reservoir. The localized perturbation approach ensures that the realizations are deformed very gradually thereby reducing the number of rejected realizations. The spatial definition of the flow regions are continually updated corresponding to the latest representation of the reservoir denoted by \( I^{\ell+1}(u) \), the reservoir model corresponding to each outer iteration. The ease of the sequential 1-D optimization procedure has an associated cost of additional inner iterations. Five inner iterations were performed for each of the 4 delineated regions and these inner iterations are embedded within the outer global minimization loop thus rendering the algorithm computationally demanding. However the computational cost is offset by the more orderly convergence characteristics of the algorithm. The objective function variations are smoother resulting in a fewer number of rejected iterations.

The production response for wells P-1 and P-3 before and after the dynamic data integration is shown in Figure 10. The figure shows that an excellent match is obtained to the well flow BHP and water-cut data after the application of the scheme. The final history matched realization is shown in Figure 11. To better understand the value added by the dynamic data, a few connectivity functions were computed. The connectivity function \( XVII \) measures the probability that all nodes on a spatial template are jointly above a target threshold. A streamline template was selected in this case. A threshold of 550md (median) was used to indicator code the data. The connectivity functions were computed along the 0 degree (N-S), 90 degree (E-W) and 45 degree (N.E-S.W) directions, Figure 12. As can be seen the initial realization constrained only to hard data displays a very different connectivity characteristic. The initial model exhibits more connectivity than the reference reservoir. The amalgamation of the dynamic data within the reservoir model serves to restrict the degree of connectivity of permeability values that are jointly below a threshold. It is to be emphasized that accurate reproduction of the connectivity characteristics implies that predictions of future performance are likely to be more accurate. The time taken for achieving the history match on the complex realistic reservoir example was 23.5 hrs on a RS-6000, P-270 Unix machine with a 4GB - 450 MHz processor.

Discussions and Conclusions
A probabilistic methodology to integrate multi-phase multi-well production data into reservoir models has been developed. The proposed algorithm is computationally efficient and could be used to integrate different types of data such as Time-lapse seismic, tracer tests, etc. The scheme has been successfully applied on several 2-D and 3-D reservoir examples. The convergence properties of the algorithm were studied in detail and this resulted in the development of rejection scheme that continuously screens random seeds to optimize favorable realizations until a history-match is achieved. The rejection scheme results in considerable computational saving in terms of unnecessary iterations spent optimizing realizations very far from the reference.

A localized perturbation approach has also been presented in this paper. The advantage of the method is that individual regions can be history-matched depending on their specific deviation from the target response. Hence the algorithm adapts itself to the extent of probability modifications required in
each of the regions of the reservoir. The other advantage of such a scheme would be in history-matching large reservoirs where development is carried out in a phased manner. The
regions of the reservoir where additional dynamic information are available could be updated while keeping the model for the other developed reservoir regions unchanged. The perturbation
scheme utilizes streamline density to decipher the different flow domains within the reservoir. Only those cells are selected for which the streamline density is greater than a
specified minimum density. This ensures that only cells that are relatively important to flow are perturbed to obtain a history match. The localized approach has been tested on a
realistic 3-D model with encouraging results.

Nomenclature

u : (x,y,z) location within reservoir
k(u) : Permeability value at location u
Z(u) : Attribute Random Variable at location u
A : The simulation event, for example permeability at a
location u
B : Combination of geological information and hard data
at neighbouring locations that influence the simulation event A
C : Production information influencing the simulation event A
f : Any flow response parameter such as Water
production rate, Well flowing bottom hole pressure, etc.
l(u, zl) : Indicator variable at location u corresponding to a
threshold zl
rD : Dynamic parameter controlling probability of
transitioning from current category to new category
over an iteration.
\Delta O : Objective function quantifying degree of deviation
from true field response.
\ell : Iteration counter
Psim : Well pressure response obtained by forward flow
simulation on current simulated realization
Ptar : Well pressure response obtained by forward flow
simulation on target realization
LPsim : Total liquid production rate obtained by forward flow
simulation on current simulated realization
LPtar : Total liquid production rate obtained by forward flow
simulation on target realization
l(u) : Indicator variable specifying the well-pair to which
location u belongs
n : Number of delineated regions in the reservoir

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Realization is history matched.

Figure 1: Flowchart of the streamline based localized perturbation approach
Objective functions v/s outer iterations

Figure 2: Target 3-D permeability model histogram.

Figure 3: Objective function behavior for indicator permeability field.

Figure 4: Histogram of reference Gaussian permeability model (md).

Figure 5: Objective function behavior for Gaussian case.

Figure 6: Objective function profile for modified rejection-sampling scheme.

Figure 7: Synclinal structure of the reference reservoir.
Figure 8: Target permeability model (md)
Figure 9: Flow regions in the top most layer

Figure 10: Response at wells P-1 and P-3 corresponding to initial, final and target models.
Figure 11: All Layers of final history-matched model (md).
Figure 12: Connectivity functions corresponding to target, initial and final realizations computed along 0, 45 and 90 degree directions.