Joint Estimation of Porosity and Saturation Using Stochastic Rock Physics Modeling
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Abstract

Porosity and saturation are fundamental sedimentary properties that affect both seismic wave propagation and appraisal development and production of an oil field. Traditionally, these parameters have been estimated from seismic impedances and velocities using Biot-Gassman theory and/or porosity-velocity relationships. However, when using theory to relate surface seismic attributes to reservoir saturation, the porosity estimation problem and its affect on the unsaturated sedimentary properties should be addressed. Similarly, the problem of porosity estimation is also affected by the hydrocarbon saturation at the target. Because bulk modulus and density are affected by both porosity and saturation of the sediment, and because the shear modulus is affected by the sediment porosities, estimating hydrocarbon saturation and reservoir porosity from seismic data is a joint estimation problem. Moreover, the uncertainties associated with saturation estimation are related to the uncertainties we have in the porosity, and vice versa.

In this paper, we present a methodology for joint porosity-saturation estimation. We show how porosities and saturations can be jointly estimated using stochastic rock physics modeling and Bayesian estimation framework. We study the interdependencies of this reservoir property estimation problem within a given lithology unit.

Methodology

Stochastic Rock Physics Modeling

Assuming a linearly elastic, isotropic earth model, the seismic response of the sediments can be completely characterized by three independent parameters: bulk modulus, shear modulus, and density. A closed-form equation for bulk and shear modulus and saturation (e.g., Biot-Gassman relations) and effective media theory (i.e., porosity-moduli relations) is derived using rock physics analysis. When sufficient data exist, such relations can be derived empirically. Note that deviation from the general theory/regression can be modeled using random error within the confidence interval of the specific relation used.

Assuming that the saturation and porosity are independent, we can draw them stochastically from a prior distribution. In this example, we use uniform distribution for porosity between 0.1 and 0.4, which means that these are the expected porosity ranges in the reservoir. Saturation is drawn similarly from a uniform distribution between 0.1 and 1 (here, we assume the residual water saturation is ~10%). Figure 1A shows the bulk modulus in the porosity-saturation space. Figure 1B shows the projection of the bulk modulus on the bulk modulus-porosity space and Figure 1C shows the projection of the bulk modulus on the modulus-saturation space. In Figure 1B, the scatter in the data can be interpreted as the results of uncertainty in the saturation, and similarly in Figure 1C, the scatter in the data can be interpreted as the result of uncertainty in the porosity. Similar calculations are done for the shear modulus and the density.

Joint Estimation Methodology

Within the framework of Bayesian inversion, we can use the following general methodology to estimate the most likely porosity and saturation given a set of seismic attributes. From stochastic rock physics modeling, we obtain the random seismic attributes ATR (ATR can be Vp, Vs, density, or any function of these three variables, e.g., P-impedance, shear impedance, and density). Then, we calculate the conditional probability of a set of seismic attributes given saturation and porosity p(ATR | φ, sw). This conditional pdf is known as the likelihood function (Kay). Next, we invert for the joint probability of the porosity and saturation using Bayes’ rule (Stengel),

\[ p(\phi, sw | ATR) = \frac{p(ATR | \phi, sw) \times p(\phi, sw)}{p(ATR)}. \]
After deriving the joint a-posteriori probability \( p(\phi, sw \mid ATR) \), we can derive the conditional probability for porosity and saturation as follows:

\[
p(\phi \mid ATR) = \int p(\phi, sw' \mid ATR)dsw',
\]

\[
p(sw \mid ATR) = \int p(\phi', sw \mid ATR)d\phi'.
\]

Note that equation (2) gives us the pdf of porosity and the pdfs of saturation, given the seismic attribute. To get an estimate of the porosity, we use the maximum a-posteriori (MAP) rule (Kayi).

![Figure 1](image1.png)

**Figure 1:** A) Bulk modulus as a function of porosity and saturation after stochastic simulation. B) Projection of A on saturation axis shows that the scatter in the data is due to uncertainty in porosity. C) Projection of A on porosity axis shows that the scatter in the data is due to uncertainty in saturation.

![Figure 2](image2.png)

**Figure 2:** Input data. A) P-impedance B) S-impedance C) Density D) Gas indicator probability
Field Example: Estimating porosity and saturation in clastic system

The input data used in our analysis are three seismic attributes: acoustic impedance, shear impedance, and density maps. These attributes are generated using hybrid inversion techniques (Benabentos et al.1) that use AVO attributes and velocity and density pseudologs generated by full waveform prestack inversion. The pay probability map we use as a lithology indicator probability map was generated using a classification scheme similar to that used by other authors (Mukerji et al.2; Avseth et al.3) and which is described in Bachrach et al.4. In this example, we chose to apply the saturation and porosity inversion to every trace where the probability of sands exceeds 20%. Figures 2A, 2B, and 2C show the P-impedance, shear impedance, and density seismic attributes we used as the three input seismic attributes for the joint estimation. Figure 2D shows the gas indicator probability map derived from well log data analysis as described in Bachrach et al.5.

The probability density functions derived in equation (2) are used as mapping functions to map the saturation and porosity from seismic attributes. However, as can be seen in Figure 1, the estimation problem is not unique. Figure 3 shows the 3D pdfs with saturation intervals of 10% and also the marginal pdfs projected on the P-impedance, S-impedance, and density space. From these pdfs we can derive not only the MAP estimator, but also the uncertainty associated with our prediction. In Figure 4, we show the results of a comparison between the saturation estimate and a saturation log. Very good agreement is visible; and particularly, zones of gas saturations higher than 65%, which are defined as risky zones for sub-commercial wells are identified and marked by arrows. Theoretical Bayesian success rates (Kay1) derived from the pdfs show that although the probability of correctly classifying the saturation within the 10% interval is only ~30% for most intervals, we can still identify the risky zones and delineate them from the pay and non-commercial values with a probability higher than 81%.

![Figure 3](image)

**Figure 3:** A) pdfs of nine saturation intervals (10% increment) in P-impedance, S-impedance, and density space. B) Marginal pdfs for P-impedance (top), S-impedance (middle), and density (bottom).

Figure 5 shows the pdfs associated with the porosity. We descretized the porosity with increments of 5% and again plot the pdfs in the 3D space of P-impedance, S-impedance, and density (Figure 5A). We also plot the marginal pdfs on each of the above axes separately in Figure 5B. Note that good separation is achievable with the three attributes. Theoretical Bayesian success rates shows that if we based our prediction...
of the data on exact P-impedance, shear impedance and density we have more than 80% probability to classify the correct porosity interval. Figure 6A shows the porosity estimation derived by the MAP rule using the pdfs in Figure 5 and the input data in Figure 2. Figure 6B shows the porosity log at trace 51, and the estimated porosity traces near the trace.

Conclusions

In this paper, we showed how joint estimation of porosity and saturation can be performed using joint Bayesian inversion. Uncertainty estimation, which reflects the non-uniqueness in the inversion problem, is an integral part of this formalism. The only assumption made with this new method is about the pay lithology. In the specific example we presented, we could differentiate potential pay from the high-risk zone with 80.1% confidence, but a saturation resolution of 10% is achievable with 30% success. In this example, we identify porosity within 91% confidence, as porosity is better resolved than saturation given a complete set of seismic attributes.

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References


Figure 5: A) pdfs of five porosity intervals in P-impedance, S-impedance, and density space. B) Marginal pdfs for P-Impedance (top), S-Impedance (middle), and density (bottom).

Figure 6: Results of MAP porosity map. A) Inline inversion B) Comparison to the well log. Good correlation between sands high-porosities and prediction is visible.