Integrating Seismic Data in Reservoir Modeling:
The Collocated Cokriging Alternative


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Abstract

The two sources of information commonly available for modeling the top of a structure, depth data from wells and geophysical measurements from seismic surveys, are often difficult to integrate. While the well data provide the most accurate measurements of depths there are rarely enough wells to permit an accurate appraisal from well data alone. On the other hand, the seismic data are generally less precise but more abundant. Two geostatistical methods, "external drift" and "collocated cokriging", are proposed to integrate the two sources of information. A case study is used to document the strengths and weaknesses of both approaches for constructing contour maps of the top structure and assessing the uncertainty on such maps through stochastic simulations.

Introduction

A major contribution of geostatistics to reservoir modeling has been the addressing of the general problem of data integration, proposing algorithms for merging data of different types, reliability, taken at different scales, into a more accurate reservoir numerical model [1-5]. As opposed to more traditional deterministic approaches, geostatistical algorithms provide an assessment of the resulting model uncertainty.

Typical of the general problem of data integration is the utilization of dense 3D seismic data and well test data for a better characterization of reservoir heterogeneities. Traditionally, seismic data are used to map major reflective horizons. Due to their increased resolution, these data are now being used to improve the petrophysical characterization of the reservoir [6,7]. As opposed to detecting major structural heterogeneities, the mapping of local petrophysical variability is a much more challenging problem calling for algorithms that can handle information taken at different scales. Indeed, the highest resolution 3D seismic data provide information still coarse when compared to core samples; although, one can argue that this coarse information better matches the scale at which flow simulation and reservoir performance forecasts are made [8,9].

To provide a focus for discussion, consider the simpler problem of mapping a specific horizon using dense 3D seismic data (travel time) and sparse control well data (horizon depth), see Figure 1. The well data provide accurate local information that can be considered as "hard". The numerous seismic locations provide a quasi exhaustive coverage, but each travel time represents some smooth local average of the depth around the CDP (Common Depth Point) location; also these seismic data should be calibrated to the control wells. One wishes to map the horizon depth with its actual spatial variability as shown by the well data, not the smoothed and possibly biased image read directly from the seismic data. Reproduction of the actual variability is much more critical when modeling permeability than when mapping the depth of a horizon. Lastly, no matter the algorithm used, the resulting estimated map is somewhat in error, and one would wish to have a measure of such potential error, a measure that can be used to evaluate the impact of such error on reservoir performance prediction.

Algorithms for mapping a primary variable from both primary (hard) and secondary (soft) data can be classified in two broad categories:

- Interpolation algorithms which yield a unique response (interpolated map), best in some sense. These interpolation algorithms are usually low-pass filters in that they tend to smooth out local details of the spatial variability of the primary variable being mapped [2]. In the best case they provide a local measure of uncertainty, e.g.,

References and illustrations at end of paper.
a kriging variance, which falls short of being a measure of joint spatial uncertainty (one that involves many estimated points simultaneously).

- **Stochastic imaging** (or simulation) techniques which provide multiple possible realizations of the unknown surface or spatial distribution, yet all such realizations honor the same original data. Typically, these stochastic algorithms are full-pass filters in that the simulated maps reproduce the full spectrum of the data spatial variability, inasmuch as that spectrum has been correctly inferred and modeled. Fluctuations between realizations (the stochastic images) provide a visual and quantitative measure of the uncertainty about the underlying phenomenon.

This paper reviews algorithms in both categories, proposing adaptations and some innovative implementations. This review is backed by comparative runs using a real data set from a reservoir overlying a salt dome in the Gulf of Mexico. All runs were produced with the GSLIB software [10].

**Notations:**

Let $z_1(u)$ be the primary variable of interest distributed over a field $A$, with $u \in A$ being the coordinate vector. In the following case study, $z_1$ represents the depth to the top of the reservoir at a 2D location $u$; integrals of $z_1(u)$ truncated by the oil-water contact would provide the gross reservoir volume. In another application, $z_1(u)$ could represent porosity at a fuzzy volume centered around location $u$, much larger than the hard data $z_1(u)$ at well locations $u \in A$; the integrals of $z_1(u)$ would be associated to the total pore volume.

Sparse but accurate (hard) sample values $z_1(u_\alpha)$, $\alpha = 1, ..., n_1$ are available at well locations $u_\alpha \in A$. In addition, seismic data $z_2(u'_\alpha)$, $\alpha = 1, ..., n_2$, of different but related nature are available at a much larger number $n_2$ of locations $u'_\alpha \in A$. Note that although the seismic CDP locations $u'_\alpha$, and the integrals of $z_1(u)$ would be associated to the total volume.

The goal of integration is to produce one or several maps for the distribution of $z_1(u)$ over field $A$ utilizing both hard data $\{z_1(u_\alpha)\}$ and soft data $\{z_2(u'_\alpha)\}$ to establish the relationship between the hard $z_1(u)$ and the soft $z_2(u'_\alpha)$.

The first category of interpolation algorithms aims at providing a unique “best” estimate of $z_1(u)$ at any node of the contouring grid. The second category of stochastic simulation algorithms would provide a distribution of $L$ images $\{z^{(l)}(u), u \in A\}$, $l = 1, ..., L$, each representing a possible representation of the underlying “true” spatial phenomenon $\{z_1(u), u \in A\}$.

Note that for both categories the goal is to map the primary variable $z_1(u)$ making use of the secondary data $z_2(u'_\alpha)$, it is not to map the secondary variable $z_2(u)$. Hence, the step of calibration of seismic data to control well data is essential: the greyscale map of travel time in Figure 1 is not directly a map of the top of the structure as some geophysicists would like to believe. Also, that calibration should be done in a real environment using field seismic data and actual well data, not in a clean sterile environment of a lab. This is particularly important when seismic data are to be used to map petrophysical properties: much too often the excellent correlation found between sonic data and lab petrophysical measurements are not borne out by field data [7].

**Regression algorithms**

Most interpolation algorithms making use of secondary information to map a primary variable are based on some form of regression of the unknown value $z_1(u)$ on the sample data, $z(u_\alpha)$, $\alpha = 1, ..., N$, the $z$-data being either of type $z_1$ (primary or well data) or of type $z_2$ (secondary or seismic).

Kriging itself is but a generalized regression algorithm whereby the unknown value is estimated by a linear combination of the $z_1$-data [11,12]:

$$z_1^*(u) = \sum_{\alpha=1}^{n_1} \lambda_1^{(1)} z_1(u_\alpha) + \sum_{\alpha=1}^{n_2} \lambda_2^{(2)} z_2(u'_\alpha)$$

From a theoretical point of view, there is no difference between kriging and cokriging. Again, calibration is limited to inference of two-point statistics relating any two values $z_1(u), z_1(u + h)$, such as the $z_1$-variogram or covariance. The weights $\lambda_1^{(1)}$ determining the regression (1) are then given by a system of normal (kriging) equations.

The apparently formidable cokriging is nothing more than an extension of that regression to include data of type different from $z_1$. For example, if $n_2$ seismic data $z_2(u'_\alpha)$ are available in addition to the $n_1$ well data $z_1(u_\alpha)$ the cokriging estimate for the primary variable at any unsampled location $u$ is:

$$z_1^*(u) = \sum_{\alpha=1}^{n_1} \lambda_1^{(1)} z_1(u_\alpha) + \sum_{\alpha=1}^{n_2} \lambda_2^{(2)} z_2(u'_\alpha)$$

From a theoretical point of view, there is no difference between kriging and cokriging. Again, calibration is limited to inference of two-point statistics relating any two values $z(u), z(u + h)$, where $z$ can be of type $z_1$ or type $z_2$, and the $(n_1 + n_2)$ weights $\lambda_1^{(1)}$ and $\lambda_2^{(2)}$ are given by a system of normal (cokriging) equations. The only difference is of practical order, that of inference and consistent modeling of four covariance functions instead of a single one in the case of kriging:

$$C_{11}(h) = \text{Cov}\{Z_1(u), Z_1(u + h)\}$$
$$C_{12}(h) = \text{Cov}\{Z_1(u), Z_2(u + h)\}$$
$$C_{21}(h) = \text{Cov}\{Z_2(u), Z_1(u + h)\}$$
$$C_{22}(h) = \text{Cov}\{Z_2(u), Z_2(u + h)\}$$

where $C_{21}(h)$ is usually assumed identical to $C_{12}(h)$.

Modern desktop workstations can easily handle the increased dimension of a cokriging system, $(n_1 + n_2)$ instead of $n_1$ for kriging. The hurdle to the wide spread use of cokriging lies in the inference and, above all, the tedious modeling of the matrix of (cross) covariances (3). Although the task of
such modeling can be made much easier using interactive and intelligent graphics software, and algorithmic developments in data integration have consisted in short-cutting the modeling of cross-covariances.

The external drift model [10,13]

This model consists of assuming that the secondary datum \( z_2(u) \) reflects, up to a linear rescaling, the behavior of some local average of the primary \( z_1 \)-values around location \( u \):

\[
\frac{1}{|V|} \int_{V(u)} z_1(u') \, du' \approx a + b z_2(u) \tag{4}
\]

where \( V(u) \) represents some volume/area centered at \( u \), with measure \( |V| \).

In terms of a random function model, the secondary variable \( z_2(u) \) is interpreted as a linear rescaling of the locally variable, hence non-stationary, expected value \( E\{Z_1(u)\} \) of the primary variable:

\[
E\{Z_1(u)\} = a + b z_2(u) \tag{5}
\]

In words, the spatial variability of the secondary variable \( z_2(u) \) is assumed to be related to local trends in the primary variable \( z_1(u) \).

Kriging with an external drift model consists of estimating by regression from the collocated \( z_1-z_2 \) data the coefficients \( a \) and \( b \), then using these estimates \( a^*, b^* \) to perform kriging from the sole primary residual data \( z_1(u) - [a + b z_2(u)] \), \( \alpha = 1, \ldots, n_1 \). This two-step procedure is collapsed into a single "universal" kriging-type system of equations [12] with the estimate being a linear combination of the \( z_1 \)-data alone:

\[
\begin{align*}
[z_1(u)]_{\text{est.drift}} - m_1 &= \sum_{\alpha=1}^{n_1} \nu_\alpha [z_1(u_\alpha) - m_1] \\
\sum_{\beta=1}^{n_1} \nu_\beta C_1(u_\beta - u_\alpha) + \mu_1 + \mu_2 z_2(u_\alpha) &= C_1(u - u_\alpha), \\
\sum_{\beta=1}^{n_1} \nu_\beta &= 1 \\
\sum_{\beta=1}^{n_1} \nu_\beta z_2(u_\alpha) &= z_2(u)
\end{align*} \tag{6}
\]

where \( m_1 = E\{Z_1(u)\} \) is the stationary mean.

Pros:
- The algorithm is extremely easy to implement. As compared to cokriging, it does not require inference of the covariances \( C_2(h) \) and \( C_12(h) \), the system is of dimension \( (n_1 + 2) \) instead of \( (n_1 + n_2) \).
- It provides, by construction, \( z_1 \)-maps that closely resemble the secondary \( z_2 \)-map, see constitutive hypothesis relation (5).

Cons:
- It provides \( z_1 \)-maps that closely resemble \( z_2 \)-maps, whether hypothesis (5) is correct or not.
- It does not capture the full \( z_1-z_2 \) spatial cross-correlation as does cokriging.
- It requires that \( z_2 \) secondary data be available at all locations \( u_\alpha, \alpha = 1, \ldots, n_1 \) of the primary data and at all nodes \( u \) being estimated, see system (6).

• Theory requires that the covariance \( C_1(h) \) used in system (6) be that of the residual \( z_1(u) - [a + b z_2(u)] \), not that of \( z_1(u) \) as used most often, nor that of the experimental residuals \( z_1(u) - [a^* + b^* z_2(u)] \).

Ideally, the assumption (4) should be based on the physics of the problem. For example, a two-way travel time \( z_2(u) \) can be reasonably associated to some local average of the reflecting horizon depth \( z_1(u) \). However, it would be more difficult to justify from basic principles a relation of type (4) between any seismic parameter \( z_2(u) \), whether travel time or seismic amplitude, and a petrophysical property \( z_1(u) \), say, porosity or directional permeability. In practice, there is never enough densely drilled well data \( z_1(u') \), \( u' \in V(u) \), to statistically check relations (4) or (5).

The collocated cokriging model

One implementation problem associated with a full cokriging approach to integration of seismic data, or of any densely sampled secondary data, is matrix instability. Indeed, the extreme proximity and the large auto-correlation of contiguous seismic \( z_2 \)-data, as opposed to the large separation distances and poor auto-correlation between primary \( z_1 \) data taken at different wells, create unstable cokriging matrices (close to singularity). In addition, the secondary datum \( z_2(u) \) collocated with the value \( z_1(u) \) to be estimated tends to screen the influence of further away secondary data.

A solution to this problem consists simply of retaining at each location \( u \) to be estimated only the collocated secondary datum \( z_2(u) \), thus making \( n_2 = 1 \) in expression (2). The estimate \( z_1^*(u) \) and corresponding simple cokriging system are written:

\[
\begin{align*}
z_1(u) - m_1 &= \sum_{\alpha=1}^{n_1} \lambda_\alpha^{(1)} [z_1(u_\alpha) - m_1] + \lambda^{(2)} [z_2(u) - m_2] \\
\sum_{\beta=1}^{n_1} \lambda_\beta^{(1)} C_1(u_\beta - u_\alpha) + \mu_1 + \mu_2 z_2(u_\alpha) &= C_1(u - u_\alpha), \\
\sum_{\beta=1}^{n_1} \lambda_\beta^{(1)} &= 1 \\
\sum_{\beta=1}^{n_1} \lambda_\beta^{(1)} z_2(u_\alpha) &= z_2(u) \\
\sum_{\beta=1}^{n_1} \lambda_\beta^{(2)} C_1(u - u_\beta) + \lambda^{(2)} C_2(0) &= C_12(0)
\end{align*} \tag{7}
\]

where \( m_1 = E\{Z_1(u)\}, m_2 = E\{Z_2(u)\} \) are the two stationary means, \( C_1(h), C_2(h), C_12(h) \) are the (cross) covariances defined in (3).

System (7) does not carry any more the small covariance values associated to highly redundant secondary \( z_2 \)-data, but it still requires inference of the cross-covariance \( C_12(h) \).

A further approximation consists of retaining for \( C_12(h) \) a Markov-type model, see Appendix to this paper.

The Markov model

Consider the Markov-type screening hypothesis:

\[
E\{Z_2(u) \mid z_1(u), z_1(u + h)\} = E\{Z_2(u) \mid z_1(u)\}, \forall z_1(u + h) \tag{8}
\]

i.e., the primary datum \( z_1(u) \) screens the influence of any other datum \( z_1(u + h) \) on the secondary collocated variable \( z_2(u) \).

Then it can be shown, see Appendix, that the cross-covariance \( C_12(h) = C_1(h) \) takes the congenial form:

\[
C_12(h) = \frac{C_12(0)}{C_1(0)} C_1(h), \forall h \tag{9}
\]
or equivalently,
\[ \rho_{12}(h) = \rho_{12}(0) \rho_1(h), \quad \forall h \]
with \( \rho_1(h) = \frac{c_{11}(h)}{c_{11}(0)} \) being the \( Z_1 \)-correlogram, \( \rho_{12}(h) = \frac{c_{12}(h)}{\sqrt{c_{11}(0)c_{22}(0)}} \) being the \( Z_1-Z_2 \) cross-correlogram, and \( \rho_{12}(0) \) being the traditional (collocated) coefficient of correlation between \( Z_1(u) \) and \( Z_2(u) \).

The Markov model (9) is particularly congenial in that it provides the cross-covariance model as a rescaled version of the primary covariance \( C_1(h) \). This model can easily be checked by running experimental covariances and cross-covariances and comparing their rescaled plots:
\[ \frac{C_{12}(h)}{C_{12}(0)} \simeq \frac{C_1(h)}{C_1(0)}, \quad \forall h \]

Collocated cokriging under a Markov model

With the Markov model, the collocated cokriging estimate and system (7) are rewritten in their standardized form:

\[
\frac{Z_1(u) - m_1}{\sigma_1} = \sum_{\alpha=1}^{n_1} \lambda^{(1)}_{\alpha} \frac{Z_1(u_\alpha) - m_1}{\sigma_1} + \lambda^{(2)} \frac{Z_2(u) - m_2}{\sigma_2} \tag{10}
\]

\[
\frac{\sum_{\beta=1}^{n_1} \lambda^{(1)}_{\beta} \rho_1(u_\beta - u_\alpha) + \lambda^{(2)} \rho_{12}(0) \rho_1(u - u_\alpha)}{\sigma_1} = \rho_1(u - u_\alpha), \quad \alpha = 1, \ldots, n_1
\]

\[
\frac{\sum_{\beta=1}^{n_1} \lambda^{(1)}_{\beta} \rho_{12}(0) \rho_1(u_\beta - u) + \lambda^{(2)} = \rho_{12}(0)}{\sigma_1} \cdot \sigma_2 = \text{Var}(Z_2(u)) \]

where \( \sigma_1^2 = \text{Var}(Z_1(u)) \), \( \sigma_2^2 = \text{Var}(Z_2(u)) \) are the two stationary variances.

Remarks

- If \( \rho_{12}(0) = 0 \), then \( \lambda^{(2)} = 0 \), and the collocated secondary datum \( Z_2(u) \) is ignored.
- If \( \rho_{12}(0) = 1 \), the system (10) is identical to a simple kriging system with \( (n_1 + 1) \) primary data \( Z_1(u_\alpha) \) and \( Z_2(u) \). The exactitude of kriging then yields the solution: \( \lambda^{(2)} = 0 \), \( \forall \alpha = 1, \ldots, n_1 \), and \( \lambda^{(2)} = 1 \). Therefore, the estimate (10) identifies the standardized collocated secondary datum, as expected.

Pros:

- The algorithm is easy to implement, as easy as kriging with an external drift model, compare system (6) and (10).
- As opposed to the constitutive hypothesis (5) of the external drift approach, the Markov model (9) can easily be checked from data.

Cons:

- Unless \( |\rho_{12}(0)| \) is large, the resulting collocated cokriging \( Z_1 \)-map may not look alike the secondary \( Z_2 \)-map. This may be considered a safeguard to overconfidence in the secondary map.
- As in kriging with an external drift, the system (10) requires the secondary variable \( Z_2(u) \) be sampled at all nodes where \( Z_1 \) is to be estimated.
- The algorithm ignores the information brought by non-collocated secondary data beyond that of the collocated datum \( Z_2(u) \).

Another major difference between the external drift-based estimate (6) and the collocated cokriging estimate (10) is that the former is not directly related to the secondary variable values, whereas the latter is. The secondary data values \( Z_2(u) \) play a role only in informing about the shape of the \( Z_1 \) trend, hence the name “external drift model”. Whereas, in the bona-fide cokriging expression (10) the secondary datum \( Z_2(u) \) influences directly the estimated \( Z_1 \)-value.

However, in our opinion, the major advantage of the collocated cokriging model is that it relies on a calibration (tuning) parameter: the correlation coefficient \( \rho_{12}(0) \), and it can be invalidated by checking the Markov model (9). There is no such tuning parameter in the external drift model, nor can it be invalidated a priori from the data.

All regression algorithms, including kriging, full cokriging, kriging with an external drift model or collocated cokriging with a Markov-type cross-covariance model, are low-pass filters that tend to yield an over-smoothed image of the actual spatial variability of the primary attribute \( Z_1 \). This smoothing may be desirable for applications involving static volumetric calculations such as mapping the top of a reservoir. It may be harmful for applications involving dynamic flow simulations such as the modeling of permeability spatial distributions: smoothing would lead to an under-representation of extreme values (conditional bias) and an erasing of patterns of spatial connectivity of such extreme values (flow barriers or flow paths if the primary variable is permeability), see [2,14]. The solution is to consider a full-pass mapping algorithm that reproduces the full spectrum (i.e., covariance) of spatial variability. Stochastic simulations are such algorithms.

Stochastic simulation algorithms (Gaussian-based)

This section is limited to a review of stochastic simulation algorithms involving the use of secondary data, more specifically seismic data complementing “hard” well data in the framework of reservoir modeling. These include two broad categories:

1. Gaussian-based simulations: the variable to be simulated is the normal score transform of the primary variable \( Z_1(u) \). The Gaussian conditional distributions are established either with a full cokriging using all secondary data or with the less demanding collocated cokriging.

2. In case the Gaussian model is proven inadequate, a non-parametric, non-Gaussian, indicator approach should be considered. This indicator approach, although more general, more powerful and also much more demanding, is not discussed in this paper, see [2,12].
Gaussian simulation with full cokriging

Provided a multivariate Gaussian model is accepted, this is the most straightforward approach. It does require though a full cokriging in the normal score transform space, with all the shortcomings attached to such full cokriging (matrix instability and tedious modeling of covariance function matrix).

More precisely, consider the normal score transforms $Y_1(u) = \phi_1(Z_1(u))$ and $Y_2(u) = \phi_2(Z_2(u))$ of the primary and secondary variables, $Z_1(u)$ and $Z_2(u)$ respectively. The covariances and cross-covariances of $Y_1(u)$ and $Y_2(u)$ are inferred and modeled from the corresponding normal score data; let them be:

$$S_1(h) = E\{ \phi_1(Z_1(u))\phi_1(Z_1(u+h)) \}$$
$$S_2(h) = E\{ \phi_2(Z_2(u))\phi_2(Z_2(u+h)) \}$$
$$S_{12}(h) = E\{ \phi_1(Z_1(u))\phi_2(Z_2(u+h)) \}$$

Recall that $E\{ \phi_1(Z_1(u)) \} = 0$, by definition of the normal score transform.

The full simple cokriging of $Y_1(u)$ using both primary data $y_1(u_\alpha), \alpha = 1, ..., n_1$ and secondary data $y_2(u_\alpha'), \alpha = 1, ..., n_2$ determines fully the Gaussian conditional cumulative distribution (ccdf) of $Y_1(u)$. The ccdf mean is the simple cokriging estimate $y_1^*(u)$ and the ccdf variance is the corresponding simple kriging variance:

$$y_1^*(u) = \sum_{\alpha=1}^{n_1} \lambda_1^{(1)} y_1(u_\alpha) + \sum_{\alpha=1}^{n_2} \lambda_2^{(2)} y_2(u_\alpha')$$
$$\sigma_{SK}^2(u) = 1 - \sum_{\alpha=1}^{n_1} \lambda_1^{(1)} S_1(u-u_\alpha) - \sum_{\alpha=1}^{n_2} \lambda_2^{(2)} S_{12}(u-u_\alpha')$$

The $(n_1 + n_2)$ weights $\lambda_1^{(1)}$ and $\lambda_2^{(2)}$ are given by a simple cokriging (normal) system.

**Pros:**
- Straightforward Gaussian theory.
- No need to model the cross-covariance $S_{12}(h)$.
- Stable kriging matrices.

**Cons:**
- Dependence on stringent joint multivariate Gaussian model with all its limitations.
- Dependence on the Markov model of type (9) for $S_{12}(h)$.
- However, this model can be checked using the original normal score data $y_1(u_\alpha)$ and $y_2(u_\alpha')$.
- Secondary data information is not retained beyond the collocated datum $y_2(u)$.

**A case-study: Mapping of a salt-dome**

This case-study relates to a reservoir in the Gulf of Mexico, where upthrust of a salt-dome has deformed the overlying strata including the top of the reservoir. The data set comprises, see Figure 1,

- depths to the top of the reservoir as measured from 20 wells; these are considered as hard data $z_1(u_\alpha), \alpha = 1, ..., n_1 = 20$.
- the two-way travel times from 3D seismic, recorded at 15,753 CDP locations; these are considered as soft data $z_2(u_\alpha'), \alpha = 1, ..., n_2 = 15,753$ informing the primary variable $z_1(u)$.

2. At node $u_i$, determine the Gaussian ccdf of $Y_1(u_i)$ from simple cokriging using all original normal score data (of both types 1 and 2) and all previously simulated values $y_1^{(i)}(u_j), j < i$ falling into a neighborhood of $u_i$.

3. Draw a value $y_2^{(i)}(u_i)$ from that ccdf and add it to the file of primary $y_1$-data.

4. Return to step (2) until all $N$ nodes have been simulated.

5. Back-transform the realization $\{y_2^{(i)}(u_i), i = 1, ..., N\}$ into the original $z_1$-space with $z_1^{(i)}(u) = \phi_1^{-1}(y_1^{(i)}(u))$.

Repeat the entire process with another random path to generate another realization $\{z_1^{(i)}(u), i = 1, ..., N\}$.

Gaussian simulation with collocated cokriging

This approach is similar to the previous one except that the full cokriging to determine the ccdf's is replaced by a less demanding collocated cokriging of type (10) using a Markov model of type (9) for the covariance $S_{12}(h)$. The mean and variance of the Gaussian ccdf of $Y_1(u)$ are:

$$y_1^*(u) = \sum_{\alpha=1}^{n_1} \lambda_1^{(1)} y_1(u_\alpha) + \lambda_2^{(2)} y_2(u)$$
$$\sigma_{SK}^2(u) = 1 - \sum_{\alpha=1}^{n_1} \lambda_1^{(1)} S_1(u-u_\alpha) - \lambda_2^{(2)} S_{12}(0)$$

**Pros:**
- Straightforward Gaussian theory.

**Cons:**
- Dependence on stringent joint multivariate Gaussian model with all its limitations.
- Secondary data information is not retained beyond the collocated datum $y_2(u)$. 

This case-study relates to a reservoir in the Gulf of Mexico, where upthrust of a salt-dome has deformed the overlying strata including the top of the reservoir. The data set comprises, see Figure 1,
The gross reservoir volume is that defined between the top surface and the oil-water contact at a depth of 5130 m.

Of the 20 wells, only 7 are located on the upthrust structure: lighter grey areas on Figure 1a corresponding to the right tail of the histogram of well data and the left tail of the histogram of travel times. The scattergram of well data vs. collocated travel times show two populations, see Figure 1d. The excellent correlation, \( \rho = -0.96 \), is not representative of the main structure (dome area). If only the seven wells of that dome area are retained, the correlation coefficient drops to \( \rho = -0.60 \).

If travel time is indeed a good indicator of the depth of the structure, any mapping of that structure using only the well data would fail to reveal the details of the dome structure, see Figure 2b.

**Ordinary Kriging**

Ordinary kriging was performed at each of the 15,753 CDP locations. At each location, all 20 well data were used. The isotropic semi-variogram model was inferred from the omnidirectional experimental semi-variogram of the 20 normal score data, see Figure 2a:

\[
\gamma(h) = K[0.05 + 0.95 \text{Gauss}_{0.5}(h) ] \tag{14}
\]

with \( \text{Gauss}_{0.5}(h) = 1 - \exp(-h^2/a^2) \) being a Gaussian model with unit sill; parameter \( a \) and effective range \( a\sqrt{3} \). The multiplicative factor \( K \) plays no role on the resulting kriging estimates; it could be identified to the variance of the 20 well depth data, i.e., \( K = 314m^2 \), see the histogram of Figure 1b. The small 5% relative nugget effect was added to avoid kriging matrix instability associated with very continuous models such as Gaussian with zero nugget effect. The Gaussian model was chosen because one expects extreme spatial continuity when mapping the top of a sedimentary structure. The experimental variogram of the original well data was too noisy to be useful. The effective range \( a\sqrt{3} = 1100m \) was borrowed from the less noisy variogram of the normal score transform of Figure 2a.

Ordinary kriging results in the smooth map of Figure 2b which fails to reveal the sharp boundaries of the dome structure seen on the travel time map of Figure 1a. Compare the histogram of the kriging estimates of Figure 2c to that of the well data of Figure 1b. Total gross reservoir volume is estimated from that kriging to be \( 84.21 \times 10^6 m^3 \).

**Kriging with external drift**

A first alternative to integrate the seismic data is kriging with a trend model linearly rescaled from the travel time data, see relation (5).

The resulting kriging map is shown on Figure 3a, with the corresponding histogram of 15,753 kriging estimates shown on Figure 3b. The variogram model used is that considered for ordinary kriging, see expression (14) and Figure 2a. All 20 well data and, correspondingly, the 20 collocated seismic data \( z_t(u_k) \) were retained for each kriging, see system (6).

The resulting estimated map delineates better the dome area shown on the seismic map of Figure 1a. Note also the significantly larger standard deviation of estimates (less smoothing effect): \( \sigma = 16.4 \) from Figure 3b, when compared to that of the ordinary kriging estimates: \( \sigma = 12.2 \) from Figure 2c.

The resulting total gross reservoir volume is \( 72.79 \times 10^6 m^3 \). 

**Collocated cokriging**

A second alternative to integrate the seismic data is kriging retaining only the travel time datum collocated with the point being estimated, and using a Markov model for the cross-covariance, see system (10).

As mentioned before, the Markov model (9) can be checked. Figure 4a shows the omnidirectional experimental cross-variogram versus the variogram obtained from the Markov model. The excellent match validates the Markov hypothesis for this case.

The resulting cokriging map is shown on Figure 4b, with the corresponding histogram of 15,753 estimated values shown on Figure 4c. The primary variogram model used is that retained for ordinary kriging, see expression (14) and Figure 2a. The correlation coefficient \( \rho_{12}(0) \) used for the Markov model (9) is that based on the seven wells intersecting the dome structure, i.e., \( \rho_{12}(0) = -0.6 \). As for the other krigings, all 20 well data were retained for each cokriging: \( n_1 = 20 \) in system (10).

The resulting estimated map delineates the dome area well as well as the external drift map of Figure 3a although with a larger smoothing effect: \( \sigma = 11.73 \) for collocated cokriging, \( \sigma = 16.44 \) for kriging with an external drift. Recall the standard deviation \( \sigma = 17.72 \) of the original 20 well data.

The resulting gross reservoir volume is \( 79.37 \times 10^6 m^3 \), a value between that given by ordinary kriging \( 84.21 \times 10^6 m^3 \) and cokriging with an external drift \( 72.79 \times 10^6 m^3 \). Some evaluation of the potential for error of these estimation values is in order.

**Conditional simulation with collocated cokriging**

Gaussian simulation conditional to both normal score well data and seismic data was implemented to generate 100 realizations of the structure top.

The variogram model fitted from the normal score transforms of the 20 well data is that of expression (14) and shown in Figure 2a. The correlation coefficient used for the Markov model of cross-covariance is \( \rho_{12}(0) = -0.6 \) corresponding to the seven wells intersecting the dome structure. Because that correlation value -0.6 is mediocre and because the primary variogram model (14) is so continuous at the origin, far away primary (well) data will carry more weight than the collocated secondary (seismic) datum. Indeed, the primary correlogram \( \rho_1(h) \) drops to under 0.6 only for distances \( |h| \) greater than 430 m. This has for consequence to dilute the influence of the seismic data and their associated smooth spatial variability.

Figure 5a and 5b gives the realizations with maximum and minimum gross reservoir volumes. Figure 5c gives the E-type estimated map, i.e., the map obtained by averaging at each of the 15,753 locations all 100 simulated values. Figure 6 gives the histogram of the 100 simulated gross reservoir volumes: the mean is the E-type estimated gross reservoir volume, \( 84.07 \times 10^6 m^3 \); the spread of that histogram provides a measure of uncertainty about that volume, e.g., the 95% probability interval for the gross reservoir volume \( V \) is:

\[
\text{Prob}(V \in [62.73, 102.44] \times 10^6 m^3) = 0.95
\]

Note the similarity in shape of the E-type estimated map of Figure 6c to the travel time map of Figure 1a.
Validation
After this case study has been done, an additional 25 wells were made available to us over the study area, see dots on Figure 7. The collocated cokriging (not simulation) algorithm (10) was repeated using all 45 wells, resulting in the map of Figure 7a to be compared to that of Figures 3a and 4b.

The updated gross reservoir volume 75.59 × 10^6 m^3 is seen to fall within the 95% probability interval provided by simulation.

Figure 8a gives the scattergram of the 25 depth estimates provided by kriging with external drift (using only 20 well data) vs. the actual values at these locations. Figure 8b gives the scattergram with, now, the 25 estimates provided by collocated cokriging (again using only 20 well data). Collocated cokriging is seen to perform slightly better.

Conclusions
In the category of Gaussian model-based algorithms, where normal score transforms of primary and secondary data are assumed to be jointly Gaussian-distributed, the collocated cokriging approximation represents an interesting practical alternative in that

1. it is fast, robust and easy to implement
2. it does not require inference of a cross-covariance model (under an additional Markov-type hypothesis that can be checked from data)
3. it can be applied in either an estimation or a simulation mode
4. it is firmly grounded in Gaussian theory.

A comparative study using well and two-way travel time seismic data for mapping of a salt dome structure indicates that collocated cokriging performs well in both

- the estimation mode, as compared to ordinary kriging and kriging with an external drift, and
- the simulation mode.

Presently available code for Gaussian sequential simulation, such as sgsim of GSLIB [10], can be easily adapted to handle the single additional equation required by collocated cokriging.

Nomenclature

\[ \gamma(h) = \text{semi-variogram} \]
\[ \lambda, \nu = \text{kriging weights} \]
\[ \rho(h) = \text{covrelogram function} \in [-1, +1] \]
\[ \sigma_1, \sigma_2 = \text{standard deviation} \]
\[ \sigma^K_2(u) = \text{kriging (estimation) variance} \]
\[ C_{ij}(h) = \text{covariance between any two random variables } i \text{ and } j \]
\[ E(\cdot) = \text{expected value} \]
\[ E[Z_2|Z_1 = z] = \text{conditional expection of } Z_2 \text{ given the value } Z_1 = z \]

References

Appendix: The Markov model for cross-covariance

Consider two stationary random functions \( Z_1(u) \) and \( Z_2(u) \). Without loss of generality, these random functions can be assumed to have zero means and unit variances; their covariances are then:

\[
\rho_1(h) = E\{Z_1(u)Z_1(u+h)\} \\
\rho_2(h) = E\{Z_2(u)Z_2(u+h)\} \\
\rho_{12}(h) = \rho_{21}(h) = E\{Z_2(u)Z_1(u+h)\} \tag{15}
\]

Next consider the two conditions:

\[
E\{Z_2(u) \mid Z_1(u) = z_1\} = \rho_{12}(0)z_1 \tag{16}
\]

i.e., the regression of \( Z_2 \) on \( Z_1 \) is linear (note that (16) is verified if \( Z_1(u) \) and \( Z_2(u) \) are jointly Gaussian-distributed), and

\[
E\{Z_2(u) \mid Z_1(u) = z\} = \rho_{12}(h) \tag{17}
\]

i.e., the collocated datum \( Z_1(u) = z \) screens the influence of any other \( z_1 \)-data on \( Z_2(u) \), then

Theorem: Conditions (16) and (17) entail the following expression for the cross-covariance:

\[
\rho_{12}(h) = \rho_{12}(0)\rho_1(h), \quad \forall h \tag{18}
\]

Proof: Let \( f_{12}(z,z') \) be the bivariate pdf of the two random variables \( Z_1(u) \) and \( Z_1(u+h) \), the cross-covariance \( \rho_{12}(h) \) is written:

\[
\rho_{12}(h) = \int \int E\{Z_2(u)Z_1(u+h) \mid Z_1(u) = z, Z_1(u+h) = z'\} f_{12}(z,z') \, dz \, dz' 
\]

\[
= \int z' \int E\{Z_2(u) \mid Z_1(u) = z, Z_1(u+h) = z'\} f_{12}(z,z') \, dz \, dz' 
\]

\[
= \int z' \int E\{Z_2(u) \mid Z_1(u) = z\} f_{12}(z,z') \, dz \, dz', \quad \text{using (17)} 
\]

\[
= \rho_{12}(0) \int z' f_{12}(z,z') \, dz \, dz', \quad \text{using (16)} 
\]

\[
= \rho_{12}(0)\rho_1(h), \quad \text{by definition of } \rho_1(h). 
\]

Figure 1: Mapping a salt dome structure
a. using two-way travel time (ms). The dots indicate well locations
b. histogram of depths measured at the wells
c. histogram of travel time data
d. scattergram of travel time and depth at 20 well locations
Figure 2: Kriging interpolation using only the 20 well data
a - semivariogram and model of 20 hard data
b - ordinary kriging estimates
c - histogram of estimates

Gross Reservoir Volume = 84.21 x 10^6 m^3

Figure 3: Kriging with external drift defined from travel time data
a - greyscale map of kriging estimates with external drift
b - histogram of kriging estimates with external drift

Gross Reservoir Volume = 72.79 x 10^6 m^3

Figure 4: Collocated cokriging
a - checking the Markov model
b - greyscale map of collocated cokriging estimates
c - histogram of collocated cokriging estimates

Gross Reservoir Volume = 79.37 x 10^6 m^3
10 Integrating Seismic Data in Reservoir Modeling

**Figure 5:** Simulation with collocated cokriging
- a - realization with maximum gross reservoir volume
- b - realization with minimum gross reservoir volume
- c - E-type estimation map

```
Gross Reservoir Volume = 109.77 \times 10^6 \text{ m}^3
```

```
Gross Reservoir Volume = 61.32 \times 10^6 \text{ m}^3
```

```
Gross Reservoir Volume = 84.07 \times 10^6 \text{ m}^3
```

Figure 6: Histogram of simulated reservoir volumes

```
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<td>0.000</td>
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\end{tabular}
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Figure 7: Collocated cokriging using 45 well data
- a - greyscale map of estimates with dots indicating the locations of the additional 25 wells
- b - histogram of collocated cokriging estimates

```
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<td>std. dev.</td>
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</table>
\end{tabular}
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Figure 8: Validation scattergrams using the late 25 well data
- a - kriging with an external drift using 20 original wells
- b - collocated cokriging using 20 original wells

```
\begin{tabular}{c|c}
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