Which are better, random or systematic acoustic surveys? A simulation using North Sea herring as an example

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This paper considers the design of acoustic surveys for estimating the mean abundance of spatially correlated populations. We examined how the choice of survey design affects the bias and precision of the sample mean as an estimator of mean abundance. Further, we investigated three different ways of estimating the error variance of the sample mean: the pooled within strata variance and two geostatistical variance estimators based on spherical and exponential models. First, we analysed four surveys to determine the spatial structure of the North Sea herring population. We developed forty different population models based on the observed amplitude and spatial distribution. We generated 1000 realizations of each model, each comprising 4000 potential transect means. Each realization was sub-sampled using eight different sampling strategies. From each realization and sampling strategy, we calculated the sample mean and three estimates of the variance of the sample mean.

The simulations show that, for surfaces with local positive correlation, more precise estimates of the surface mean can be obtained using stratified random or systematic sampling than simple random sampling. The best strategy considered here was (a) a systematic survey with a geostatistical variance estimator, when the main objective is to obtain the most precise estimator of abundance, (b) a stratified random survey, with two transects per strata, and a pooled variance estimator, when an important objective of the survey is to obtain a good estimate of the variance of the abundance estimator.

Key words: acoustic surveys, abundance estimation, survey strategy, surveys, variance estimation.

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Introduction

Many marine surveys for fish stock estimation are carried out annually or more often by a number of countries. The costs of each survey can easily reach £200 000, so it is important that the survey is designed and the data analysed to maximize the information that can be obtained about the stock. An area of current debate is the appropriate survey methodology for spatially correlated populations. The problems of data analysis for such populations have been discussed in various reports of the ICES Working Group on Methods of Fish Stock Assessment (e.g. Anon., 1990, 1992). The choice of design has been examined in less detail.

This study considers the design of acoustic surveys for estimating the mean abundance of spatially correlated populations. In particular, we focus on populations with spatial properties similar to those of North Sea herring: namely, a local positive autocorrelation, a small-scale random component, and a large-scale autocorrelation or trend.

We first consider the bias and error variance of the sample mean as obtained from eight survey designs: a simple random survey, five increasingly stratified random surveys, a systematic survey with a random starting point, and a systematic survey with a centred starting point. From a series of simulations, we show that precision increases with increasing stratification, with the systematic surveys generally giving the most precise estimator. This is in broad agreement with other studies; see, for example, Venrick (1978) and Lenarz and Adams (1980) on sampling chlorophyll and rockfish populations respectively. Cochran (1977) also reported that, based on a few examples, systematic surveys compared favourably in precision with stratified random surveys.
Bellhouse (1977) showed that, for a very general class of two-dimensional covariance structures, a type of systematic sampling is optimal within three subclasses of design. Ripley (1981) considered the error variance of the sample mean for a stationary isotropic random processes. He showed that, if there is strong local positive correlation, both stratified random and systematic surveys should do well compared to a simple random survey. Further, unless there is strong periodicity in the data, systematic sampling should perform better than stratified random sampling.

We also consider ways of estimating the error variance. It is well known that the “classical” variance estimator for stratified random surveys - which we call the “pooled within-strata variance” - is unbiased when there are two or more samples per strata, whatever the population under study (Cochran, 1977; Thompson, 1992). However, no equivalent estimator exists for systematic surveys or stratified random surveys with one sample per strata. One approach for these designs is to combine adjacent strata and use the pooled, within-strata variance. Another approach is to use a model-based estimator. For example, Heilbron (1978) developed variance estimators for systematic surveys when the population is generated by a Gaussian stationary serial correlation model. Here, we examine two model-based geostatistical variance estimators, derived from exponential and spherical variogram models but not constrained to Gaussian or stationary data distributions, and contrast them with the pooled within-strata variance.

We take a super-population view of sampling throughout. That is, we consider the distribution of estimators averaged over both population realisations and with the randomisation of sampling units. It is, of course, well known that stratified random surveys with two or more samples per strata allow for design-unbiased abundance and variance estimators (unbiased over randomisation of sampling units, whatever the underlying population). They therefore have great appeal, being free from any model assumptions. However, we are particularly interested in the distribution of both abundance and variance estimators over a range of plausible underlying models (see Smith (1990) and Thompson (1992, chapter 10) for discussions on design-, model-, and unconditional-unbiasedness).

We only consider stratified random surveys in which (a) the survey area is divided into adjacent strata of equal size and (b) there are an equal number of samples per strata. More efficient survey designs can often be obtained if strata, and allocation of sampling effort to strata, can be based on environmental variables such as depth, or on previously observed stock distributions (e.g. Smith and Gavaris, 1993). Here we address the choices for survey design once such strata have been identified or where such relationships cannot be established.

There have been some attempts to assess strategies for acoustic surveys. Nickerson and Dowd (1977) concluded that zig-zag strategies are optimal, but their results are difficult to assess as they do not explicitly describe their simulations. Vorobyov (1983) also concluded that a zig-zag pattern is optimal. However, his simulations implied different ship speeds for different survey strategies, raising doubts about his results. In addition, his simulations do not include any spatial auto-correlation. Kimura and Lemberg (1981) used a model with randomly located circular fish schools. Their conclusions are heavily dependent on the selection of a usable transect length, equivalent to the number of samples. This varied with survey strategy, as inter-transect sections were excluded for parallel strategies. The spatial auto-correlation in their model is limited by the size of the schools. Jolly and Hampton (1990) stated that some form of transect randomization is desirable, otherwise no valid estimate of sampling error can be made from a single survey unless the population is randomly distributed. They described a stratified random survey with a two-stage sampling procedure. None of these studies consider the spatial auto-correlation of fish abundance in detail, nor directly address the precision of variance estimators. Aglen (1989) investigated the precision of many surveys by subsampling and computes an empirical relationship between precision and effort. He found considerable differences between populations with different spatial characteristics. However, he did not consider the precision of variance estimates from a single survey. We consider it essential to investigate the distribution of both abundance and variance estimators and that the spatial distribution of the population is an integral part of this investigation.

In our simulations, the survey was considered as a series of point samples on a line. This corresponds to parallel transects taken perpendicular to one edge of the survey area, with the transect abundance being the observation. For most depths and echo-sounder transmission rates, transmissions overlap and very little, if any, water along the transect is missed. Independent samples (from each pulse volume) tend to come from highly skewed distributions but, as they are usually collected at a rate of about 18 000 per minute, this component of sampling error in the mean will be negligible for even quite short transects. Thus, ignoring navigation errors and temporal effects, the transect sum can be regarded as an exhaustive sample without error. Petitgas (1993) has argued that this is a reasonable procedure and Jolly and Hampton (1990) used the same procedure for variance estimation. The estimation process therefore reduces to one dimension. However, it must be remembered that navigation errors and temporal changes in fish distribution make it impossible to replicate a transect without error; some small-scale measurement error is therefore essential in any model.
Methods

Examination of data from herring surveys

The statistical properties of four acoustic surveys of North Sea herring were investigated in one dimension by examining transect abundance. The amplitude distribution of transect abundance was estimated by normalising the abundances in each year and then combining them over years (Fig. 1).

Spatial autocorrelation in data can be investigated by plotting variograms, which show how the (half) average squared differences between observations change with their distance apart (Cressie, 1991). Between-transect variograms, based on transect abundances, were plotted for each survey, and all surveys combined, on both the original and a log-transformed scale. More information on spatial correlation at short range was obtained by plotting along-transect variograms, using 2.5 nm sections of transects as point values. The variograms for all surveys combined are shown in Figures 2 and 3.

The variograms on the original scale are difficult to interpret. There is some evidence of local positive correlation and of either trend or large scale spatial autocorrelation. The log-transformed data shows clear local positive autocorrelation with a range of eight to 10 transects. However, this range is much greater than that suggested by the untransformed data. In our simulations, we generally assumed a range of 2.5 transects, but we also carried out a series of simulations with ranges between 0.75 and 20 transects. There is some indication of a trend, since the southern half of the area contained lower abundance than the northern part for all four surveys. However, it is not possible to distinguish conclusively between trend and large scale spatial correlation so both possibilities were considered in the simulations. In conclusion the stock showed:

- Short-scale measurement error (a nugget effect in geostatistical terminology) contributing 0–50% of the total variance;
- Local positive correlation, with a range between one and ten transects, contributing 40–100% of the total variance;
- Trend or large scale autocorrelation, contributing 0–50% of the total variance.

Simulation procedures

To investigate all these possible characteristics, a range of surfaces with differing statistical properties was
simulated. The surfaces contained local positive correlation, a short-scale random component, and a non-stationary or trend component. The local positive correlation was generated using an auto-regressive function chosen to give spatial autocorrelation similar to the herring surveys, with a range of 2.5 transects. However, ranges from 0.75 to 20 transects were also considered. The non-stationary or trend component was generated in three ways: a simple random walk, a linear trend, and a cosine trend of 3/4 of a wavelength from $-\pi/4$ to $5\pi/4$. The relative proportions of these components were difficult to establish from the survey data, so were varied to examine the sensitivity of the conclusions to a wide range of situations. Finally, surface amplitude values were modified so that the simulated amplitude distribution was similar to the distribution observed on the four surveys.

For each set of conditions, 1000 surfaces of 4000 locations were generated. From each surface, 40 samples were taken according to each of eight different sampling strategies. This sampling intensity is similar to that of the herring surveys. Further, the choice of 4000 locations ensures that any finite population correction is less than 1%. The sample mean was calculated and the error variance estimated in three different ways. Details of the surface generation method are given in Appendix A.

The eight sampling strategies were:

1. Forty transects randomly located in one stratum (40/1)
2. Twenty transects randomly located in each of two strata (20/2)
3. Ten transects randomly located in each of four strata (10/4)
4. Five transects randomly located in each of eight strata (5/8)
5. Two transects randomly located in each of twenty strata (2/20)
6. One transect sampled in each of 40 strata (1/40)
7. Forty transects with systematic spacing and a random start (Sys-R and)
8. Forty transects with systematic spacing and a centred start (Sys-Cent)

The strata boundaries were located systematically with equal spacing throughout the area.

Let the survey area be denoted by X and consist of a line of $M=4000$ equi-spaced locations, labelled $x_m$, $m=1, \ldots, M$. For each realised (simulated) surface, let $s_m$ be the surface value at $x_m$. An example of a surface is shown in Figure 4, with a set of samples from a systematic centred survey. The true mean value of the surface is then:

$$E = \frac{1}{M} \sum_{m=1}^{M} s_m$$  \hspace{1cm} (1)

For each survey, let $x_{ij}$ be the location of the jth transect in the ith strata (with the systematic surveys regarded as one transect in each of 40 strata). Note that the suffix m denotes all possible locations throughout the survey area, while the suffixes i and j denote specific locations sampled in a particular survey. Let $s_{ij}$ be the surface value at $x_{ij}$, J be the number of transects in each strata and I the number of strata, and N be the total number of transects (I J), which for these simulations was 40.

The sample mean in the ith strata is:

$$\bar{s}_i = \frac{1}{J} \sum_{j=1}^{J} s_{ij}$$  \hspace{1cm} (2)

and the overall sample mean is:

$$\bar{s} = \frac{1}{N} \sum_{i=1}^{I} \sum_{j=1}^{J} s_{ij}$$  \hspace{1cm} (3)

The mean abundance was estimated by the overall sample mean:

$$\bar{E} = \bar{s}$$  \hspace{1cm} (4)

The error variance $\text{Var}(\bar{E} - E)$ was estimated in three ways:

1. “Pooled within strata variance”:

$$\text{Var}_p(\bar{E} - E) = \frac{1}{N(N-1)} \sum_{i=1}^{I} \sum_{j=1}^{J} (s_{ij} - \bar{s}_i)^2$$  \hspace{1cm} (5)

For surveys with only one transect per strata, adjacent strata were combined in pairs. Note that this is a simplification of the usual variance estimator for a stratified random survey because we considered only equal sized strata with equal allocation and because the usual finite population correction (less than 1%) was ignored.

2. Geostatistical estimation variance using a spherical model with nugget, $\text{Var}_{gs}(\bar{E} - E)$, fitted by an iterated least squares procedure. The expression for the variance is given in Matheron (1971):

$$\text{Var}_{gs}(\bar{E} - E) = 2\gamma(X, x_{ij}) - \gamma(X, X) - \gamma(x_{ij}, x_{ij})$$  \hspace{1cm} (6)

where:

$$\gamma(X, x_{ij}) = \frac{1}{M^2 \sum_{m \neq n} \gamma(x_m, x_m)}$$
$$\gamma(X, x_{ij}) = \frac{1}{MN \sum_{i \neq j} \gamma(x_{ij}, x_{ij})}$$  \hspace{1cm} (7)

$$\gamma(x_{ij}, x_{ij}) = \frac{1}{N^2 \sum_{i' \neq j'} \gamma(x_{ij}, x_{ij})}$$

and where $\gamma$ is the variogram giving the half mean squared difference between observations at locations x and x+d as a function of their separation d:
In geostatistical terms, $a$ is known as the nugget, $a+b$ the sill, and $R$ the range. Geostatistical estimation variance using an exponential model with nugget, $\text{Var}_p(\bar{E} - E)$, fitted by an iterated least squares fitting procedure. The variance is estimated using equations 6 and 7 above and with $\gamma$, the variogram:

$$
\gamma(d) = \begin{cases} 
0, & d = 0 \\
a + b(1.5(1.5 - 0.5d^2)), & 0 < d < R; \\
a + b, & d \geq R. 
\end{cases} 
$$

(8)

In geostatistical terms, $a$ is known as the nugget, $a+b$ the sill, and $R$ the range.

3. Geostatistical estimation variance using an exponential model with nugget, $\text{Var}_p(\bar{E} - E)$, fitted by an iterated least squares fitting procedure. The variance is estimated using equations 6 and 7 above and with $\gamma$, the variogram:

$$
\gamma(d) = \begin{cases} 
0, & d = 0; \\
a + b(1 - e^{-d/R}), & d > 0. 
\end{cases} 
$$

(9)

Here, $a$ is the nugget, $a+b$ the sill, and $R$ the range, although the effective range is approximately $3R$.

The iterated least squares procedure for both geostatistical estimators used the experimental variogram derived from samples pooled in 39 distance bins and weighted by the number of samples per bin squared. The experimental variogram is given by:

$$
\gamma(d) = \frac{1}{2N(d)} \sum_{i,j}^{N(d)} [s_{ij} - s_{i,j}]^2 
$$

(10)

where $d$ and $N(d)$ are the mean distance between sample pairs and the number of sample pairs within each distance bin, respectively, and where the summation is over all sample pairs such that $|x_i - x_j|$ lie in the bin. The variogram parameter estimates were indistinguishable from the fit to the cloud of 780 sample pairs obtained from 40 data values (not put into bins) and computationally much faster.

The sample variance estimator

$$
\text{Var}_s(\bar{E} - E) = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j=1}^{i} (s_{ij} - \bar{s})^2 
$$

(11)

was also investigated. However, as expected for populations with local positive correlation, it was always

Figure 4. A simulated surface (on a 4000 location base line) with statistical properties similar to North Sea herring showing sample locations and values from a systematic centred sampling strategy.
The statistical properties of the abundance estimator, \( \hat{\theta} \), were investigated by comparing the simulated estimates, \( \hat{\theta}_{\text{sim}} \) say, with the simulated abundances \( E_{\text{sim}} \). Negligible bias was found for all sampling strategies and simulated surfaces. The true error variance, \( \text{Var}(\hat{\theta} - E) \), was estimated by:

\[
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\]

\[\text{Var}(\hat{\theta} - E) \]


Figure 5. The mean variogram for a surface with 50% positive correlation, 35% nugget and 15% trend components (1000 simulated surface), for random walk (solid), cosine (dotted) and linear (dashed) trend functions.

The variogram of the complete simulated surfaces was also computed to check on the statistical properties of the generated surfaces.

The mean, median, and 90% intervals of the abundance and variance estimators were estimated for each sampling strategy and each set of simulated surfaces. The lower 90% limit, the median, and upper 90% limit were obtained by sorting the 1000 simulated estimates and selecting those in locations 50, 500 and 950, respectively.

Results

To check the simulations, variograms for each surface type were calculated from the complete surface of all 1000 realizations. Examples are shown in Figure 5 for three surface types, each with 35% nugget and 50% positive autocorrelation with a range of 2.5 transects, and one of three different trend components (15%).

An example of the cloud of variogram point pairs, derived from the systematic sample and surface in Figure 4, is shown in Figure 6. The experimental variogram is also shown, together with the fitted exponential and spherical variograms that are almost indistinguishable.

Precision of the sample mean

For a given range of local positive autocorrelation, the relationship between the true error variance \( \text{Var}(E - E) \) and sampling strategy was similar for all surface types. Figure 7 summarizes the results for a range of 2.5 transects; for ease of presentation, the error variances are averaged over the surface types, for each type of non-stationary component. The simple random sample, on the left of the figure, has the highest error variance. The error variance decreases monotonically as the degree of stratification increases. This agrees with theory: for an infinite population, a stratified random survey with equally sized strata and an equal sampling allocation to strata always has a smaller (or equal) error variance than a simple random survey (Matheron, 1971, 1989). In our simulations, the error variance decreases further for systematic random and further still for systematic centred surveys; however, no general conclusion can be drawn for systematic surveys. The error variances with different ranges of positive autocorrelation show a similar shape to those in Figure 7. However, the longer the range, the greater the reduction in error variance with increasing stratification. In all cases studied here, the error variance was a minimum for systematic strategies.

Distribution of variance estimators

For a given range of local positive autocorrelation, the effects of sampling strategy on the distributions of the three variance estimators were similar for all surface types. Figure 8 summarizes the results for a range of 2.5 transects; the results are combined over all surface types. The pooled variance estimator \( \text{Var}_{p}(\hat{\theta} - E) \) is unbiased from the simple random survey to two transects. For strategies with only one transect per strata, the estimator is positively biased. The 90% interval narrows with increasing stratification and reaches a minimum when there are two transects per strata.

Both geostatistical estimators give similar results. The means of these estimators are close to the true error variance for all sampling strategies. The 90% interval is largest for the simple random survey, narrows to a minimum at two transects per strata, and widens slightly for strategies with one transect per strata. The exponential model performs better than the spherical model for systematic strategies. This is because it more correctly matches the local positive autocorrelation used in the simulations. However, it is not possible to determine which model is more appropriate for the herring surveys. Generally, the geostatistical estimators give a wider spread of estimates than the pooled variance estimator.
However, the geostatistical estimators involve an iterated least squares fitting procedure, arbitrarily chosen models, and no non-stationary components; alternative fitting procedures or theoretical variograms might improve on these results. For example Cressie (1991), suggests a fourth root transform of squared differences yields a more robust estimator of the variogram. Cressie also discusses alternative fitting procedures.

Scatter plots of the pooled variance estimates and both geostatistical estimates obtained from the same realisations are shown in Figures 9 and 10. The two geostatistical estimators are very similar, suggesting that the choice between them is not critical here. The geostatistical estimators have a longer tail than the pooled variance estimator for the simple random survey.

The minimum 90% interval for variance estimation occurs at two transects per strata for 34 of the 40 simulated surface types. However, with a highly correlated spatial distribution (an autocorrelation range greater than thirteen transects or 30% of the area) and a small nugget, the minimum 90% variance interval occurs with a strategy of one transect per strata. Conversely, with a more random distribution (an autocorrelation range of about one transect) or a large nugget (greater than 50% of the variance) the minimum 90% variance...
interval is found in more random strategies. There is an important point to be noted here. In general, increasing stratification increases the precision of the abundance estimator but reduces the degrees of freedom available for estimating that precision. In terms of our variance estimators, this means that with increasing stratification the mean variance decreases and the 90% variance interval relative to the mean variance increases. Thus, the optimal survey strategy for variance estimation depends on the balance between these two effects. For most of our populations, the autocorrelation structure was such that the reduction in mean variance, with increasing stratification, dominated until two transects per strata. As populations became more autocorrelated, the reduction in mean variance was more dominant, leading to a minimum 90% variance interval with systematic strategies. Conversely for a more random population the reduction in mean variance was less dominant, leading to a minimum 90% variance interval at more random strategies. Clearly, with a completely random population, simple random sampling is the optimal strategy.

The effects of periodicity were examined in further simulations by including a non-stationary cosine component with varying amplitude and wavelength. When the periodicity is greater than the transect spacing, and accounts for less than 50% of the variance, the minimum 90% interval still occurs at two transects per strata. Simulations were also carried out with both Gaussian and more highly skewed amplitude distributions. The general conclusions of a minimum 90% variance interval at two transects per strata were not sensitive to these changes in amplitude distribution.

To summarize, Figure 11 shows the mean 90% intervals for the abundance estimator, the pooled variance estimator, and the two geostatistical estimators, over all the simulations. The pooled variance estimator is only shown for strategies for which it is unbiased.

The simulations are based on data from four surveys and the absolute levels of variance are derived directly from these data. However, the relative precision of the variance and abundance estimators are particularly interesting. Ignoring all other sources of error in the surveys of North Sea herring, such as variation in target

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Figure 8. The true error variance (thick line) and the median and 90% intervals of estimated variance for three variance estimators, pooled (●), spherical model (△) and exponential model (■).

Figure 9. Scatter plot of variance estimates by two geostatistical models for simple random strategy (light squares) and two transects per strata strategy (dark squares).

Figure 10. Scatter plot of variance estimates by geostatistical spherical model and pooled variance for simple random strategy (light squares) and two transects per strata (dark squares).
strength, these simulations suggest that the 90% interval of the abundance estimator is 15% of its mean. In contrast, the 90% interval of the variance estimator from the most favourable survey method (two transects per strata) is 130% of its mean. The precision of the variance estimator should be borne in mind when using variance estimates to compare methods or as a stratifying variable.

Choice of survey strategy

The final choice of strategy depends on the objectives of the study. The decision is often dominated by the relative importance of the precision of the estimator of mean abundance and the ability to estimate that precision. It is straightforward to estimate mean abundance and its precision from a stratified random survey with two or more samples per strata, since “classical” sampling theory can be applied (e.g. Cochran, 1977) and no assumptions need be made about the underlying population. As the survey design becomes more systematic, variance estimation becomes more complicated, since assumptions about the underlying population must be introduced.

The results described here provide information on how survey strategy affects the bias and precision of both estimators of abundance and of the corresponding error variance. One way of using this information is to construct a decision surface showing the optimal survey strategy, given user defined weights for the allocation of effort to improve the precision of either the abundance or the variance estimator. Since the absolute levels of abundance and variance are very different, a function is required that expresses the relative change in the precision of the abundance and variance estimators with changing sampling strategy.

One such function is given by the normalized 90% interval $I_{\text{best}}/I_s$ for both abundance and variance. Here, $I_{\text{best}}$ is the smallest 90% interval over all strategies and estimators and $I_s$ is the 90% interval for a particular strategy $s$. For abundance, $I_{\text{best}}$ would correspond to the systematic centred survey. For variance, $I_{v\text{best}}$ would correspond to the pooled variance estimator with two transects per strata; further, $I_s$ would correspond to the pooled variance estimator for surveys with two or more transects per strata, and one of the geostatistical estimators for surveys with one transect per strata. The best strategy for any chosen survey objective, or weight regime, could then be the survey that maximised.

$$W_a I_{a\text{best}} + W_v I_{v\text{best}}$$

where $W_a + W_v = 1$. $I_{a\text{best}}$ is the 90% interval for abundance using strategy $s$, $I_{a\text{best}}$ is the minimum 90% interval for abundance for all strategies, $I_{v\text{best}}$ is the 90% interval for variance using strategy $s$, and $I_{v\text{best}}$ is the minimum 90% interval for variance for all strategies.

The decision surface based on our simulations is shown in Figure 12. Thus, for example, if 80% of effort is allocated for estimating abundance, and 20% for estimating variance, then a systematic survey is the optimal strategy. If 56–100% of effort is allocated for estimating abundance, and 0–44% for estimating variance, the best strategy is a systematic survey with variance estimated using a geostatistical model. Conversely, if 0–56% of effort is allocated for estimating abundance, and 44–100% for estimating variance, the best strategy is two random transects per strata and pooled variance estimation.

Conclusions

1. Estimating abundance. These simulations show that, for surfaces with local positive correlation, more precise estimates of the surface mean can be obtained using stratified random or systematic sampling than with uniform random sampling. The increase in precision depends on the relationship between spatial correlation, sampling intensity, and the region to be sampled. However, the improvement in precision is dominated by the relationship between the sample spacing and the range of correlation. Stratified or systematic sampling is likely to give the greatest benefit if the sampling intensity is greater than the scale of the spatial correlation. For an infinite population, stratified random sampling with equally sized strata, and an equal allocation of samples to strata, always gives an error variance lower or equal to that of a simple random sample.
2. Estimating abundance and variance. If a stock distribution is similar to those examined in this paper, and if mean abundance is estimated by the sample mean, then the best strategies, of those considered here, are:

(a) systematic strategies, when the main aim is to improve the precision of the abundance estimator;
(b) two transects per strata, when the main aim is to improve the estimate of the error variance;
(c) with an equal allocation of effort for both abundance and variance estimation, the optimal strategy is two transects per strata; however, there is little to choose between this strategy and the strategies with one transect per strata (either systematic or random).

The statistical models examined included various proportions of a short scale random process, local positive correlation with varying range, and three different non-stationary components. Simulations with different amplitude distributions and periodic spatial distributions suggest that the general conclusions hold under a wide range of conditions. Except for spatial distributions dominated by short-scale random effects, or strong positive correlation with a long range, the conclusions do not depend on the non-stationary component, nor on the exact combination of the different components.

It should be remembered that stratification on covariables, and alternative effort allocations, may improve on the stratified random surveys described above.

3. Precision of the variance estimators. The precision of the variance estimators, relative to their means, is substantially less than the precision of the abundance estimator. In this study the 90% confidence interval of the variance estimator was 8.6 times that of the 90% interval of the abundance estimator.

4. Bias. Precision is not the only criteria with which to assess sampling strategies. Bias is also important. Except for the systematic centred survey, all the sampling strategies here give unbiased estimators of mean abundance. The systematic centred survey will generally give biased results if there is a trend in the underlying process (e.g., fish located preferentially), although this bias was negligible in all our simulations. Because of the possibility of bias, in our view, the systematic random strategy is preferred to the systematic centred strategy.
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References

Appendix A
Surface generation method
The aim of the surface generation method was to provide amplitude and spatial distributions similar to those observed in the herring survey data. For this purpose, the following model was developed by the authors to simulate $M=4000$ surface values $s_m$ at locations $x_m$, (for the autoregressive functions 8000 values were generated and the last 4000 used):

$$s_m = T \left( \sqrt{a}p_m + \sqrt{b}q_m + \sqrt{c}r_m + K \right) \tag{1}$$

where:
- the $p_m$ are independent and normally distributed with mean 0 and variance 1, giving the very short scale random or nugget component
  $$p_m \sim N(0,1) \tag{2}$$
- the $q_m$ are an autoregressive series giving the local positive autocorrelation: the $q_m$ are defined by:
  $$q_m = a q_{m-1} + N(0,d) \tag{3}$$
  where $a$ is varied to give an autocorrelation range of 0.75 to 20 transects and $d$ is chosen so that the sample variance of the $q_m$ within the surface is 1.
- the $r_m$ are a non-stationary component generated by one of three different methods.
  (a) a simple random walk:
    $$r_m = r_{m-1} + N(0,d) \tag{4}$$
    where $d$ is chosen so that the sample variance of the $r_m$ within the surface is 1;
  (b) a linear trend
    $$r_m = f + g x_m \tag{5}$$
    where $f$ and $g$ are chosen so that the $r_m$ have a sample mean of 0 and a sample variance of 1;
  (c) a cosine trend
    $$r_m = h \cos \left( \frac{3x_m \pi}{2M} + \frac{\pi}{4} \right) \tag{6}$$
    where $h$ is chosen so that the $r_m$ have a sample variance of 1.

The coefficients $a$, $b$, and $c$ in equation 1 are chosen so that $a+b+c=1$ and are varied to combine the $p_m$, $q_m$ and $r_m$ in 10 different proportions covering the following ranges:
- Nugget effect component 0–50%
- Non-stationary or trend component 0–50%
- Local positive correlation 40–100%
- Range of local positive correlations 0.75–20 Transects

The constant $K$ in equation 1 is set equal to 35. This provides a surface with all the required spatial properties except that the amplitude distribution is approximately

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Gaussian. The function $T$ was found experimentally, and transforms the amplitude distribution to one similar to the observed distribution:

$$T(z) = \begin{cases} 
  z & \text{for } z > 40 \\
  0.00321 z^2 + 0.4542 z + 16.66 & \text{for } z \leq 40
\end{cases}$$ (7)